RAK0018 ROMAN RAKUS PŘ. Č. 6 ~~PAP~~ PAPÍR Č. 1

1) $y' + y \tan x = \tan x$

$y' = -y \tan x + \tan x$

I $y' = -y \tan x$

$\frac{y'}{y} = -\tan x$

$\int \frac{dy}{y} = \int -\tan x \, dx$

$\ln y = \ln|\cos x| + c$

$y_H = \cos x$ pro $c = 0$

$y_H = C \cdot \cos x$ $C \in \mathbb{R}$

II $y_p = C(x) \cdot \cos x$

$y_p' = -y_p \tan x + \tan x$

$C'(x) \cos x - C(x) \sin x = -C(x) \cos x \tan x + \tan x$

$C'(x) - C(x) \tan x = -C(x) \tan x + \frac{\tan x}{\cos x}$

$C'(x) = \frac{\sin x}{\cos^2 x}$

$C(x) = \int \frac{\sin x}{\cos^2 x} dx = \begin{vmatrix} u = \sin x & u' = \cos x \\ v = \tan x & v' = \frac{1}{\cos^2 x} \end{vmatrix} = \frac{\sin^2 x}{\cos x} - \int \sin x \, dx = \frac{\sin^2 x}{\cos x} + \cos x = \frac{1}{\cos x}$

$y_p = 1$

$y = C \cdot \cos x + 1$ $C \in \mathbb{R}$

2) $y'' + y = \sin x \qquad y(\pi) = y'(\pi) = 1$

$\lambda^2 + 1 = 0 \qquad y = e^{\lambda x} = e^{ix} = e^{0}(\cos x + i \sin x) \begin{matrix} \nearrow y_1 = \cos x \\ \searrow y_2 = \sin x \end{matrix}$

$\lambda_1 = i$

$\lambda_2 = -i$

$y_H = c_1 \cos x + c_2 \sin x \qquad c_1, c_2 \in \mathbb{R}$

$y_p = C_1(x) \cos x + C_2(x) \sin x$

$C_1'(x) \cos x + C_2'(x) \sin x = 0$

$C_1'(x)(-\sin x) + C_2'(x) \cos x = \sin 2x$

$C_1'(x) = -\dfrac{C_2'(x) \sin x}{\cos x}$

$-\left(\dfrac{\sin x}{\cos x}\right)(-\sin x)\left(C_2'(x)\right) + C_2'(x) \cos x = \sin 2x$

$\dfrac{\sin^2 x}{\cos x} C_2'(x) + C_2'(x) \cos x = \sin 2x$

$C_2'(x)\left(\dfrac{\sin^2 x}{\cos x} + \cos x\right) = \sin 2x$

$C_2'(x) \cdot \dfrac{1}{\cos x} = \sin 2x$

$C_2'(x) = \sin 2x \cos x$

$C_2(x) = \int \sin 2x \cos x \, dx = \int 2 \sin x \overbrace{\cos x \cos x}^{\cos^2 x} dx = \left| \begin{matrix} \cancel{u = \sin x} & \cancel{v' = \cos x} \\ \cancel{u' = \cos x} & \cancel{v = \sin x} \end{matrix} \right| = \left| \begin{matrix} s = \cos x \\ ds = -\sin x \, dx \end{matrix} \right| =$

$= \int 2 s^2 (-1) \, ds = -\dfrac{2}{3} s^3 = -\dfrac{2}{3} \cos^3 x$

$C_1'(x) = \dfrac{-\sin 2x \cos x \sin x}{\cos x} = -2 \sin x \cos x \sin x = -2 \sin^2 x \cos x$

$C_1(x) = \int -2 \sin^2 x \cos x = \left| \begin{matrix} s = \sin x \\ ds = \cos x \, dx \end{matrix} \right| = \int -2 s^2 \, ds = -\dfrac{2}{3} s^3 = -\dfrac{2}{3} \sin^3 x$

$y = y_H + y_P = c_1 \cos x + c_2 \sin x - \dfrac{2}{3} \sin^3 x \cos x - \dfrac{2}{3} \cos^3 x \sin x$

$1 = c_1 \cos \pi + c_2 \sin \pi - \dfrac{2}{3} \sin^3 \pi \cos \pi - \dfrac{2}{3} \cos^3 \pi \sin \pi$

$1 = -c_1$

$c_1 = -1$